Que. Explain Fresnel's half period zones, show that the radii of the half period zones are proportional to the square not of natural numbers.

Ans **Fresnel's half period zones :→**

Light wave travelling in space set the hypothetical ether particles in vibration. the continuous locus of ether particles in the same phase of vibration is called " Wave front". According to Huygen's principle each point on wave front sends out secondary wavelets. Fresnel assumed that these wavelets interfere and produce a resultant intensity at a point due to wave-front, Fresnel divided the wavefront

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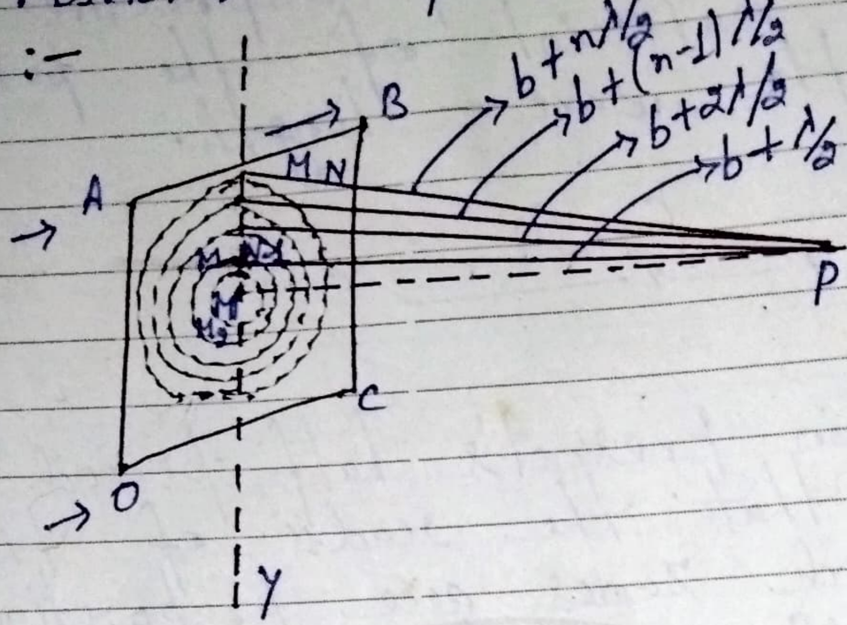
into a number of zones called fresnel's half period zones.

fig :-



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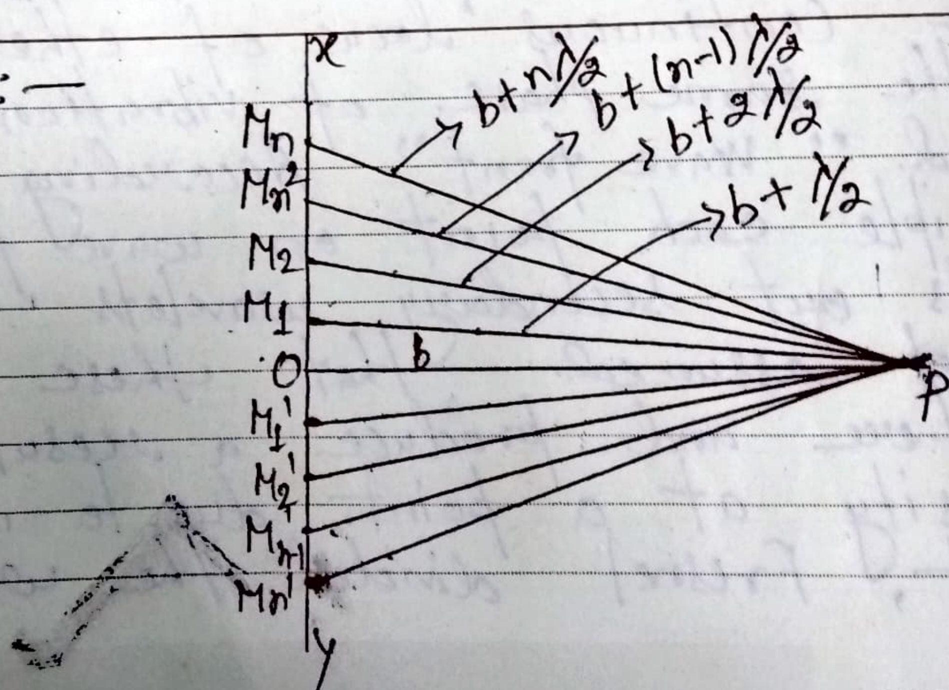

Let ABCD be a plane wave-front (perpendicular to the plane of the paper) of monochromatic light of wavelength λ, travelling from left to right. Let P be an internal point at which the effect at the entire wavefront is desired. OP is perpendicular drawn from P to the wavefront. Let $OP = b$. In order to find the resultant intensity at P due to the wave-front, by Fresnel's method, the wave-front is divided into a number of concentric half period zones called Fresnel's zones and then the effect of all the zones at point P is found.

With P as centre and radii equal to $b+\lambda/2$, $b+2\lambda/2$ .... etc. we draw a series of spheres on the wave-front. Thus cutting the wave front into annular strips or zones. The sections of these spheres by the plane wave front are concentric circles having common centre O and radii OM_1, OM_2 ---- OM_{n-1}, OM_n etc.

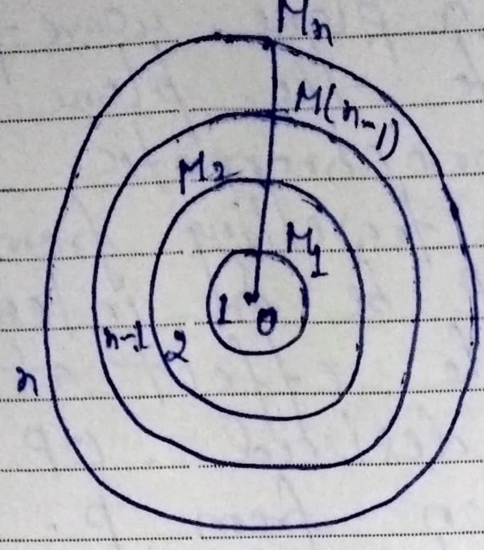

Fig :- I

The secondary wavelets from any two consecutive zones reach P with a path difference $\lambda/2$ or time difference half period. that is why the zones are called half period zones. the area of the innermost (first) circle is called first half period zones, the annular area between the first and second circles is called second half period zone and so on. thus, the annular area between (n-1)th and nth circle is called nth half period zone.

The point O is called the pole of the wavefront with respect to point P. A fresnel half

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period zones with respect to an external point P is a thin annular zone or a thin strip of the primary wave-front surrounding the point O such that the distance of its outer inner edge from O difference by $\lambda/2$.

Radii of half period zones: →

The radius of first half period zones.

$$r_1 = OM_1 = \sqrt{\{(M_1P)^2 - (OP)^2 - (OP)^2\}}$$

$$= \sqrt{\{(b+\frac{\lambda}{2})^2 - b^2\}}$$

$$= \sqrt{\{\cancel{b^2} + \frac{\lambda^2}{4} + b\lambda - \cancel{b^2}\}}$$

$$= \sqrt{(\frac{b^2}{4} + b\lambda)}$$

$= \sqrt{(b\lambda)}$ approx as ~~bλ~~ $b >> \lambda$.

The radius of second half period zones.

$$r_2 = OM_2 = \sqrt{\{(M_2P)^2 - (OP)^2\}}$$

$$= \sqrt{\{(b + \frac{2\lambda}{2})^2 - b^2\}}$$

$$= \sqrt{\{\cancel{b^2} + \lambda^2 + 2b\lambda - \cancel{b^2}\}}$$

$$= \sqrt{(2b\lambda)} \quad \text{approx.}$$

Similarly the radius of nth half period zone.

$$r_n = OM_n = \sqrt{\{(PM_n)^2 - (OP)^2\}}$$

$$= \sqrt{\{(b + \frac{n\lambda}{2})^2 - b^2\}}$$

$$= \sqrt{(\cancel{b^2} + \frac{n^2\lambda^2}{4} + bn\lambda - \cancel{b^2})}$$

$$= \sqrt{(\frac{n^2\lambda^2}{4} + bn\lambda)}$$

$= \sqrt{(bn\lambda)}$ approx.

i.e. $r_n \propto \sqrt{n}$

Thus, we see that the radii of half period zones are nearly proportional to the square roots of the natural numbers.

proved